LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2024**

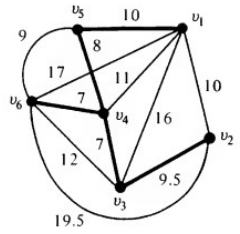
UMT 5601 – GRAPH THEORY

Date: 21-11-2024 Dept. No. Max. : 100 Marks

Time: 09:00 am-12:00 pm

	SECTION A - K1 (CO1)	
	Answer ALL the Questions -	**(10 x 1 = 10)**
1.	**Answer the following**	
a)	Define Subgraph.	
b)	Describe Unicursal graph.	
c)	State the eccentricity of a vertex in the graph.	
d)	Define Cut-set.	
e)	Describe Region.	
2.	**MCQ**	
a)	The number of vertices of odd degree in a graph is (i) Prime (ii) Composite (iii) Odd (iv) Even	
b)	An Euler graph G is arbitrarily traceable from vertex v in G if and only if every _____ in G contains v. (i) Path (ii) Walk (iii) Circuit (iv) Subgraph	
c)	Every connected graph has _____ spanning tree. (i) At most one (ii) At least one (iii) Exactly one (iv) Exactly two	
d)	Every cut-set in a connected graph G must contain at least one _____ of every spanning tree of G. (i) branch (ii) chord (iii) edge (iv) vertex	
e)	In any simple connected planar graph with f regions, n vertices and e edges ($e > 2$), which of the following inequality holds. (i) $e \leq 3(n-2)$ (ii) $e \geq 3(n-2)$ (iii) $e < 3(n-2)$ (iv) $e > 3(n-2)$	
	SECTION A - K2 (CO1)	
	Answer ALL the Questions	**(10 x 1 = 10)**
3.	**Fill in the blanks**	
a)	An open walk in which no vertex appears more than once is called _____.	
b)	A simple graph in which there exists an edge between every pair of vertices is called a _____.	
c)	The number of branches in any spanning tree of the graph G is known as _____.	
d)	The vertex connectivity of any graph G can never exceed the _____ of G.	
e)	The spherical embedding of every planar 3-connected graph is _____.	
4.	**True or False**	
a)	If two graphs have same number of vertices and edges then they are called isomorphic graphs.	
b)	A sufficient condition for a simple graph G with n vertices to have a Hamiltonian circuit is that the degree of every vertex in G be at least $n/2$.	
c)	If there is a real number associated with each edge of the graph G, then the graph is known as weighted graph.	

d)	A connected graph is said to be separable if its vertex connectivity is two.
e)	Any simple planar graph can be embedded in a plane such that every edge is drawn as a straight line segment.

	SECTION B - K3 (CO2)
Answer any TWO of the following	**(2 x 10 = 20)**
5.	Nine members of a new club meet each day for lunch at a round table. They decide to sit such that every member has different neighbours at each lunch. How many days can this arrangement last? Write all the arrangements.
6.	Prove that a simple graph with *n* vertices and *k* components can have at most $(n-k)(n-k+1)/2$ edges.
7.	What is Konigsberg bridge problem? Explain in detail how Euler solved it using a graph.
8.	List all the operations performed on the graph and explain any three of the operations with suitable graphs.
	SECTION C – K4 (CO3)
Answer any TWO of the following	**(2 x 10 = 20)**
9.	Prove that a spanning tree *T* of a given weighted connected graph *G* is a shortest spanning tree of *G* if and only if there exists no other spanning tree of *G* at a distance of one from *T* whose weight is smaller than that of *T.*
10.	Prove that with respect to a given spanning tree *T,* a chord *ci* for some *i,* that determines a fundamental circuit Γ occurs in every fundamental cut-set associated with the branches in Γ and in no other.
11.	Prove that a connected planar graph with *n* vertices and *e* edges has *e-n+2* regions.
12.	Using Prim's algorithm, estimate the shortest spanning tree of the graph given below. 
	SECTION D – K5 (CO4)
Answer any ONE of the following	**(1 x 20 = 20)**
13.	(a) Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degree. (b) In a complete graph with n vertices, prove that there are (n — l)/2 edge-disjoint Hamiltonian circuits, if n is an odd number greater than 3. (12+8)
14.	List the properties of trees and prove any four of them.
	SECTION E – K6 (CO5)
Answer any ONE of the following	**(1 x 20 = 20)**
15.	Construct a proof to defend the statement "the ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets."
16.	Defend Kuratowski's first and second graphs are non-planar.

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